1 Simplifying Boolean Expressions

In order to simplify Boolean Expressions we need to make use of Boolean Algebra’s Basic Laws, Distributive Laws, the Absorption Laws and DeMorgan’s Laws.

Basic Laws

| $0 + 0 = 0$ | $0 + 1 = 1$ | $1 + 1 = 1$ | $1 + 0 = 1$ |
| $0 \cdot 0 = 0$ | $0 \cdot 1 = 0$ | $1 \cdot 1 = 1$ | $1 \cdot 0 = 0$ |
| $A + 0 = A$ | $A + 1 = 1$ | $A + A = A$ | $A + A = 1$ |
| $A \cdot 0 = 0$ | $A \cdot 1 = A$ | $AA = A$ | $AA = 0$ |

Distributive Laws

$A(B + C) = AB + AC$, $(A + B)C = AC + BC$

Absorption Laws

$A + AB = A$

$A + \bar{A}B = A + B$

DeMorgan’s Laws

$\bar{AB} = \bar{A} + \bar{B}$

$\bar{A+B} = \bar{A}\bar{B}$

Example: Simplify the Boolean expression $C + CD$

$C + CD = C(1 + D) = C(1) = C$ . Also straight from the first Absorption Law we should see that $C + CD = C$

Example: Simplify the Boolean expression $A(B + AB) + AC$

$A(B + AB) + AC = AB + AC = A(B + C)$
**Example:** Simplify the Boolean expression \(AB\overline{C} + A\overline{B}\overline{C}\)

\[AB\overline{C} + A\overline{B}\overline{C} = AB(C + \overline{C}) = AB(1) = AB\]

**Example:** Simplify the Boolean expression \(AB + B(B + \overline{C}) + \overline{B}C\)

\[AB + B(B + \overline{C}) + \overline{B}C = AB + BB + B\overline{C} + \overline{B}C = AB + \overline{B}C = B + C\]

**Example:** Simplify the Boolean expression \(\overline{C}F + F(A + \overline{B}) + C\)

\[\overline{C}F + F(A + \overline{B}) + C = C + \overline{C}F + F(A + \overline{B}) = C + F + F(A + \overline{B}) = C + F\]

2 Boolean Expressions from Truth Tables

For this we will make use of either of 2 methods:

1. Sum-of-Products(SOP)
2. Product-of-Sums(POS)

**Sum-of-Products(SOP)**

To write a Boolean Expression using SOP we only need to do the following:

1. Find all rows of the truth table that gives 1 as output.
2. Write the inputs of each row found in step 1 as a product.
3. Take the products obtained in step 2 and form a sum.

**Example:** Given the following truth table write a corresponding Boolean Expression.

<table>
<thead>
<tr>
<th>Input A</th>
<th>Input B</th>
<th>Input C</th>
<th>Output X</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
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<td>0</td>
</tr>
</tbody>
</table>
Here row 3 and row 4 has 1 as output. For row 3 we have $A = 0$, $B = 1$ and $C = 0$ which corresponds to the product $\bar{A}B\bar{C}$. Row 4 has $A = 0$, $B = 1$ and $C = 1$ which corresponds to the product $\bar{A}BC$. Therefore $\bar{A}B\bar{C} + \bar{A}BC$ is a Boolean Expression obtained from the truth table. Looking at the expression we see that it can be simplified to give $\bar{A}B$. How?

**Product-of-Sums (POS)**

To write a Boolean Expression using POS we only need to do the following:

1. Find all rows of the truth table that gives 0 as output.
2. Write the inputs of each row found in step 1 as a sum. 
   (for $A = 0$ we write $A$ and for $A = 1$ we write $\bar{A}$)
3. Take the sums obtained in step 2 and form a product.

**Example**: Given the following truth table write a corresponding Boolean Expression using POS.

<table>
<thead>
<tr>
<th>Input $A$</th>
<th>Input $B$</th>
<th>Input $C$</th>
<th>Output $X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
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<td>1</td>
</tr>
</tbody>
</table>

1. Rows 5 and 7 have 0 as output.

2. Corresponding sums are $\bar{A} + B + C$ and $\bar{A} + \bar{B} + C$

3. Expression obtained by POS is $(\bar{A} + B + C)(\bar{A} + \bar{B} + C)$

We will now simplify the above expression.

$$(\bar{A} + B + C)(\bar{A} + B + C) = \bar{A}A + \bar{A}B + \bar{A}C + \bar{A}B + B\bar{B} + BC + \bar{A}C + \bar{B}C + CC$$

$$= \bar{A} + \bar{A}B + \bar{A}C + \bar{A}B + BC + \bar{B}C + C$$
\[
= A + \bar{A}C + \bar{A}B + BC + C \\
\]
\[
= A + C + \bar{A}B = A + \bar{A}B + C = A + B + C
\]

3 Logic Circuits from Boolean Expressions

**Example:** Draw a logic circuit for the Boolean Expression $\bar{A}B$.
We see that we are dealing with 2 gates here. A NOT gate for A and an AND gate for $\bar{A}B$.

**Example:** Draw the logic circuit for $AB + C$.

**Example:** Draw a logic circuit for $X = (A + B + C)(AB + C)$.

Suppose we had simplified the Boolean Expression first, what would our circuit look like?