Notes on Boolean Algebra

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1 Simplifying Boolean Expressions

In order to simplify Boolean Expressions we need to make use of Boolean Algebra's Basic Laws, Distributive Laws, the Absorption Laws and DeMorgan's Laws.

Basic Laws

$0+0=0$	$0+1=1$	$1+1=1$	$1+0=1$
$0 \cdot 0=0$	$0 \cdot 1=0$	$1 \cdot 1=1$	$1 \cdot 0=0$
$A+0=A$	$A+1=1$	$A+A=A$	$A+\bar{A}=1$
$A \cdot 0=0$	$A \cdot 1=A$	$AA=A$	$A\bar{A}=0$

Distributive Laws
$A(B+C)=AB+AC$, $(A+B)C=AC+BC$

Absorption Laws
$A+AB=A$
$A+\bar{A}B=A+B$

DeMorgan's Laws
$\overline{AB}=\bar{A}+\bar{B}$
$\overline{A+B}=\bar{A}\bar{B}$

Example: Simplify the Boolean expression $C+CD$

$C+CD=C(1+D)=C(1)=C$. Also straight from the first Absorption Law we should see that $C+CD=C$

Example: Simplify the Boolean expression $A(B+AB)+AC$

$$A(B+AB)+AC=AB+AC=A(B+C)$$

Example: Simplify the Boolean expression $A\bar{B}C + A\bar{B}\bar{C}$

$$A\bar{B}C + A\bar{B}\bar{C} = A\bar{B}(C + \bar{C}) = A\bar{B}(1) = A\bar{B}$$

Example: Simplify the Boolean expression $AB + B(B + \bar{C}) + \bar{B}C$

$$AB + B(B + \bar{C}) + \bar{B}C = AB + BB + B\bar{C} + \bar{B}C$$

$$= AB + \underbrace{B + B\bar{C}}_{B} + \bar{B}C = \underbrace{AB + B}_{B} + \bar{B}C = B + C$$

Example: Simplify the Boolean expression $\bar{C}F + F(A + \bar{B}) + C$

$$\bar{C}F + F(A + \bar{B}) + C = C + \bar{C}F + F(A + \bar{B}) = C + \underbrace{F + F(A + \bar{B})}_{F} = C + F$$

2 Boolean Expressions from Truth Tables

For this we will make use of either of 2 methods:

1. Sum-of-Products(SOP)
2. Product-of-Sums(POS)

Sum-of-Products(SOP)

To write a Boolean Expression using SOP we only need to do the following:

1. Find all rows of the truth table that gives 1 as output.
2. Write the inputs of each row found in step 1 as a product.
3. Take the products obtained in step 2 and form a sum.

Example: Given the following truth table write a corresponding Boolean Expression.

Input A	*Input* B	*Input* C	*Output* X
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

Here row 3 and row 4 has 1 as output. For row 3 we have $A = 0$, $B = 1$ and $C = 0$ which corresponds to the product $\bar{A}B\bar{C}$. Row 4 has $A = 0$, $B = 1$ and $C = 1$ which corresponds to the product $\bar{A}BC$.
Therefore $\bar{A}B\bar{C} + \bar{A}BC$ is a Boolean Expression obtained from the truth table. Looking at the expression we see that it can be simplified to give $\bar{A}B$. How?

Product-of-Sums (POS)
To write a Boolean Expression using POS we only need to do the following:

1. Find all rows of the truth table that gives 0 as output.
2. Write the inputs of each row found in step 1 as a sum.
(for $A = 0$ we write A and for $A = 1$ we write $\bar{A}$)
3. Take the sums obtained in step 2 and form a product.

Example: Given the following truth table write a corresponding Boolean Expression using POS.

Input A	*Input* B	*Input* C	*Output* X
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

1. Rows 5 and 7 have 0 as output.
2. Corresponding sums are $\bar{A} + B + C$ and $\bar{A} + \bar{B} + C$
3. Expression obtained by POS is $(\bar{A} + B + C)(\bar{A} + \bar{B} + C)$

We will now simplify the above expression.
$(\bar{A}+B+C)(\bar{A}+\bar{B}+C) = \bar{A}\bar{A}+\bar{A}\bar{B}+\bar{A}C+\bar{A}B+B\bar{B}+BC+\bar{A}C+\bar{B}C+CC$

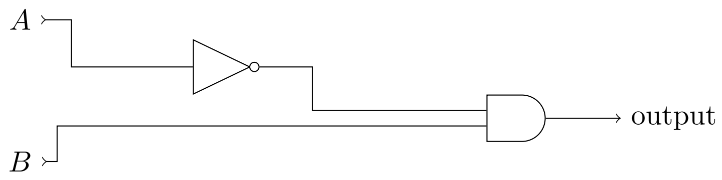
$$= \bar{A} + \bar{A}\bar{B} + \bar{A}C + \bar{A}B + BC + \bar{B}C + C$$

$$= \underbrace{A + \bar{A}C}_{A+C} + \bar{A}B + \underbrace{BC + C}_{C}$$

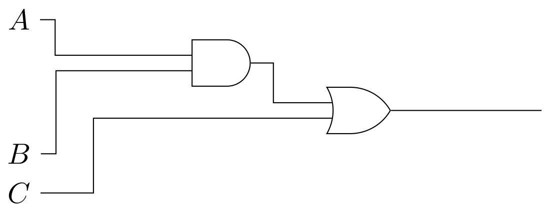
$$= A + C + \bar{A}B = \underbrace{A + \bar{A}B}_{A+B} + C = A + B + C$$

3 Logic Circuits from Boolean Expressions

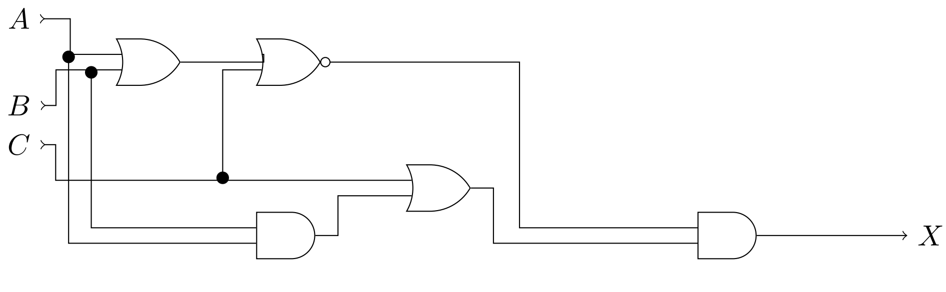
Example: Draw a logic circuit for the Boolean Expression $\bar{A}B$.
We see that we are dealing with 2 gates here. A NOT gate for A and an AND gate for $\bar{A}B$.



Example: Draw the logic circuit for $AB + C$.



Example: Draw a logic circuit for $X = (\overline{(A + B) + C})(AB + C)$



Suppose we had simplified the Boolean Expression first, what would our circuit look like?