Applications of the Determinant

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Abstract

In this short note two-dimensional applications of the determinant is presented. The note is actually part of a lecture delivered by the author at the Faculty of Technology of the University of Guyana. The hope is that these notes will give the beginning engineering student a better understanding of basic uses of determinants.

1 Finding the area of a triangle given the coordinates of its vertices

For the triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ the area is given by $A = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Example Find the area of the triangle with vertices (0,0), (5,0) and(5,4).

$$A = \pm\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 5 & 0 & 1 \\ 5 & 4 & 1 \end{vmatrix} = \pm\frac{1}{2}\begin{vmatrix} 5 & 0 \\ 5 & 4 \end{vmatrix} = 10$$

2 Determining if 3 points are collinear

If three points are collinear (lying on the same line), then any triangle formed by them must have zero area. Therefore we must have,

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Example Determine if (0,0), (5,0) and(5,4) are collinear.

$$\begin{vmatrix} 0 & 0 & 1 \\ 5 & 0 & 1 \\ 5 & 4 & 1 \end{vmatrix} = 20 \neq 0 \Rightarrow \text{the points are not collinear.}$$

3 Finding the equation of a line given two points

The equation of a line with 2 known points (x_1, y_1) and (x_2, y_2) is given by

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

Example Find the equation of the line that passes through (1,1) and (1,5).

$$\begin{vmatrix} x & y & 1 \\ 1 & 1 & 1 \\ 1 & 5 & 1 \end{vmatrix} = 0 \Rightarrow \; x\begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} - y\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 1 & 5 \end{vmatrix} = 0$$

$\Rightarrow -4x + 4 = 0 \Rightarrow x = 1$ is the equation of the line.

4 Finding the equation of a circle given 3 points on the circle

The equation of a circle with 3 known points (x_1, y_1) , (x_2, y_2) and (x_3, y_3) is given by

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ x_1^2+y_1^2 & x_1 & y_1 & 1 \\ x_2^2+y_2^2 & x_2 & y_2 & 1 \\ x_3^2+y_3^2 & x_3 & y_3 & 1 \end{vmatrix} = 0$$

Example Find the equation of the circle that passes through (4,4),(4,-4) and (-4,0).

This means that we must have,

$$(x^2+y^2)\begin{vmatrix} 4 & 4 & 1 \\ 4 & -4 & 1 \\ -4 & 0 & 1 \end{vmatrix} - x\begin{vmatrix} 32 & 4 & 1 \\ 32 & -4 & 1 \\ 16 & 0 & 1 \end{vmatrix} + y\begin{vmatrix} 32 & 4 & 1 \\ 32 & 4 & 1 \\ 16 & -4 & 1 \end{vmatrix} - \begin{vmatrix} 32 & 4 & 4 \\ 32 & 4 & -4 \\ 16 & -4 & 0 \end{vmatrix} = 0$$

This gives,
$(x^2+y^2)[4(-4)-4(-4)-4(8)]-x(-128)+y(0)-[32(-16)-32(16)+16(-32)] = 0$
$\Rightarrow -64(x^2+y^2) + 128x + 1536 = 0 \Rightarrow x^2 + y^2 - 2x - 24 = 0$ is the equation of the circle.

If we need the equation of the circle that exposes the center and radius then we need to complete the square. We have,

$$x^2 - 2x + y^2 = 24$$

which gives $(x-1)^2 + y^2 = 25$ on completing the square. The radius of the circle is therefore 5 and the center is (1,0).

5 Problems

1. Find the equation of the line through (2,-1) and (5,3).
2. Find the equation of the circle through (0,1), (1,0) and (3,0).
3. Determine whether (2,0), (5,3) and (6,5) are collinear.
4. Find the area of the triangle with vertices (1,0), (2,3) and (3,6).
5. Find the area of the rectangle determined by (2,3) and (6,-4) using determinants.

6 References

[1] Arthur Copeland. *Geometry,Algebra and Trigonometry by Vector Methods*,1962
[2] Roland Larson, Bruce Edwards, *Elementary Linear Algebra*, 1991