EMT121 - EXAM I

Laurel Benn

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1. Find all antiderivatives of $f(x) = x(x^2 + 2)$.
   Soln: $f(x) = x(x^2 + 2) = x^3 + 2x$.
   All antiderivatives are therefore represented by $\frac{x^4}{4} + x^2 + C$.

2. If f is a function satisfying $f''(x) = 2$, $f(0) = -4$, and $f(2) = 0$. Find f.
   Soln: $f''(x) = 2 \Rightarrow f'(x) = 2x + C \ \Rightarrow f(x) = x^2 + Cx + K$.
   Now $f(0) = -4 \Rightarrow 0^2 + C(0) + K = -4 \Rightarrow K = -4$.
   $f(2) = 0 \Rightarrow 2^2 + C(2) - 4 = 0 \Rightarrow 2C = 0 \ \Rightarrow C = 0$.
   Hence $f(x) = x^2 - 4$.

3. For some choice of $f(x)$, a and b , the quantity

$$\lim_{n\to\infty} \sum_{i=1}^{n} ((\frac{i}{n})^2) \cdot \frac{1}{n}$$

   is equal to $\displaystyle\int_a^b f(x)\,dx$. Find a suitable such $f(x)$, a and b.
   Soln: $f(x) = x^2$, $a = 0$, $b = 1$ are suitable choices.

4. Evaluate.

   (a) $\displaystyle\int_1^4 \sqrt{4x}\,dx \ = 2\int_1^4 \sqrt{x}\,dx = \ \frac{4}{3}\left[x^{\frac{3}{2}}\right]_1^4 = \frac{4}{3}(8-1) = \frac{28}{3}$

   (b) $\displaystyle\int (x + \frac{1}{x})^2\,dx \ = \int (x^2 + 2 + \frac{1}{x^2})\,dx = \ \frac{x^3}{3} + 2x - \frac{1}{x} + C$

   (c) $\displaystyle\int \frac{3x^2 + x}{x}\,dx \ = \int (3x + 1)\,dx = \frac{3x^2}{2} + x + C$

   (d) $\displaystyle\int \frac{\sin(1-4x)}{3}\,dx \ = \frac{\cos(1-4x)}{12} + C$

   (e) $\displaystyle\int \frac{dx}{5x-3} = \ \frac{1}{5}\ln(5x-3) + C$

5. Consider the region R, in the first quadrant, bounded above by $y = 3x$ and below by $y = x^2$.

(a) Find the area of R.

$$A = \int_0^3 (3x - x^2)\,dx \;= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3 \;= \frac{27}{2} - \frac{27}{3} = \frac{27}{6} = \frac{9}{2}$$

(b) Find the volume of the solid that is obtained by rotating R about the y-axis.

$$V = \int_0^3 2\pi x(3x - x^2)\,dx \;= 2\pi \int_0^3 (3x^2 - x^3)\,dx \;= 2\pi \left[x^3 - \frac{x^4}{4}\right]_0^3$$
$$= 2\pi \left[27 - \frac{81}{4}\right] = 2\pi \cdot \frac{27}{4} = \frac{27\pi}{2}$$

(c) Find the volume of the solid that is obtained by rotating R about the x-axis.

$$V = \pi \int_0^3 [(3x)^2 - (x^2)^2]\,dx = \pi \int_0^3 (9x^2 - x^4)\,dx = \pi \left[\frac{9x^3}{3} - \frac{x^5}{5}\right]_0^3$$
$$= \pi \left[81 - \frac{3 \cdot 81}{5}\right] = 81\pi \left[1 - \frac{3}{5}\right] = 81\pi \cdot \frac{2}{5} = \frac{162\pi}{5}$$