EMT111 - Solutions for Practice Test II

Laurel Benn

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1. Find the domain and range for $f(x) = \sqrt{1-x}$:
   Solution: Here the domain is all x such that $1 - x \geq 0$. $\Rightarrow (-\infty, 1]$ is the domain.The range is all the possible $f(x)$ values. This is of course $[0, +\infty)$.

2. If $f(x) = x^2 + 1$ and $g(x) = 1 - x$.Find the following:
   (a) $(f \circ g)(x)$
   Solution:$(f \circ g)(x) = f(g(x)) = f(1-x) = (1-x)^2+1 = x^2-2x+2$
   (b) $g \circ f$
   Solution:$(g \circ f) = g(f(x)) = g(x^2 + 1) = 1 - (x^2 + 1) = -x^2$.
   (c) a function h such that $(g \circ h)(x) = x$
   Solution:$(g \circ h)(x) = x \Rightarrow g(h(x)) = x$. Now for this to be true, h must be the inverse of g. But $g^{-1} = 1 - x$. $\Rightarrow h = 1 - x$ is the function needed.
   (d) all values of x such that $f(g(x)) = g(f(x))$.
   Solution: from (a) and (b) we must have $-x^2 = x^2 - 2x + 2$.
   $$\Rightarrow 2x^2 - 2x + 2 = 0$$
   $$\Rightarrow x^2 - x + 1 = 0$$
   Now the last equation has no solution in the reals. Therefore we conclude that there is no x such that $f(g(x)) = g(f(x))$.

3. Find the minimum value of the function $f(x) = x^2 + 2x + 1$.
   Solution: $c - \frac{b^2}{4a} = 1 - \frac{2^2}{4} = 0$ is the minimum value.

4. Is $f(x) = x + 11$ one-to-one? Explain.
   Solution: a function f is 1-1 if $f(a) = f(b) \Rightarrow a = b$. Suppose $f(a) = f(b)$. Then $a + 11 = b + 11 \Rightarrow a = b + 11 - 11 = b$. Therefore f is one-to-one.

5. Find the inverse of the following $f(x) = 3x - 1$.
   Solution:$f^{-1}(x) = \frac{x+1}{3}$.

6. Solve for x.$\frac{x}{x+2} < -1$.
   Solution: Adding 1 to both sides we get $\frac{x}{x+2} + 1 < 0$

$$\Rightarrow \frac{x + x + 2}{x + 2} < 0$$
$$\Rightarrow \frac{2x + 2}{x + 2} < 0$$

   Now -1 makes the numerator zero and -2 makes the denominator zero. We therefore split the number line into the intervals $(-\infty, -2)$, $(-2, -1)$ and $(-1, +\infty)$ for investigation.By inspection we find that $x \in (-2, -1)$ is the solution.

7. Solve. $\log_2(x - 7) = 3$
   Solution:$\log_2(x - 7) = 3 \Rightarrow x - 7 = 2^3 = 8 \Rightarrow x = 8 + 7 = 15$.

8. Find $\sin\theta$ if $\sec\theta = 25/7$ and $\frac{3\pi}{2} < \theta < 2\pi$.
   Solution: $\sin^2\theta = 1 - \cos^2\theta = 1 - 49/625 = 576/625 \quad \Rightarrow \sin\theta = -\sqrt{\frac{576}{625}} = -\frac{24}{25}$. The minus sign was taken since θ is in quadrant IV.

9. Find $\sin 135^o$ using a sum or difference formula.
   Solution: $\sin 135^o = \sin(90^o + 45^o) = \sin 90^o \cos 45^o + \sin 45^o \ \cos 90^o$
   $= 1 \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot 0 = \ \frac{\sqrt{2}}{2}$

10. Evaluate the following limits:

    (a) $\lim_{x\to\infty} \frac{x^2 + 3}{4x^2 - 1}$
        Solution: Dividing by x^2 we get
        $\lim_{x\to\infty} \frac{1 + \frac{3}{x^2}}{4 - \frac{1}{x^2}} = \frac{1}{4}$

(b) $\lim_{x\to\infty} \frac{x^3+4}{5x^2-1}$
Solution: Dividing by x^2 we get
$\lim_{x\to\infty} \frac{x+\frac{4}{x^2}}{5-\frac{1}{x^2}} = \frac{\infty+0}{5-0} = \infty$

(c) $\lim_{x\to\infty} \frac{x^2-2}{3x^3+4}$
Solution:Dividing by x^3 we get
$\lim_{x\to\infty} \frac{\frac{1}{x}-\frac{2}{x^3}}{3+\frac{4}{x^3}} = \frac{0-0}{3-0} = 0$

(d) $\lim_{x\to\infty} \frac{8x+1}{\sqrt{4x^2+x}}$
Solution: Dividing by x we get
$\lim_{x\to\infty} \frac{8+\frac{1}{x}}{\sqrt{4+\frac{1}{x}}} = \frac{8}{2} = 4$

(e) $\lim_{x\to 2} \frac{x^2-3x}{x+1}$
Solution: Here we have a somewhat easy direct substitution
$\lim_{x\to 2} \frac{x^2-3x}{x+1} = \frac{2^2-3\cdot 2}{2+1} = -\frac{2}{3}$

(f) $\lim_{x\to 1} \frac{x^2+x-2}{x^2-1}$
Solution:This one needs factorization since direct substitution will give $\frac{0}{0}$. Hence we have the following:
$\lim_{x\to 1} \frac{x^2+x-2}{x^2-1} = \lim_{x\to 1} \frac{(x-1)(x+2)}{(x-1)(x+1)} = \lim_{x\to 1} \frac{x+2}{x+1} = \frac{3}{2}$

(g) $\lim_{x\to 1} \left(\frac{1}{x^2} - \frac{1}{x}\right)$
Solution: Well this gives $1-1=0$ obviously.

(h) $\lim_{x\to 0^-} \left(x^3 - \frac{1}{x^2}\right)$
Solution:
This gives us $\lim_{x\to 0^-} x^3 - \lim_{x\to 0^-} \frac{1}{x^2} = 0 - \infty = -\infty$.

(i) $\lim_{x\to -\infty} \frac{x(x-3)}{7-x^2}$
Solution: Expand the numerator and divide by the highest power

in the denominator to get

$$\lim_{x\to-\infty} \frac{1-\frac{3}{x}}{\frac{7}{x^2}-1} = -1$$

11. Verify the following identities:

  (a) $\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$
  Solution:
  $\tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \frac{\sqrt{\frac{1-\cos\theta}{2}}}{\sqrt{\frac{1+\cos\theta}{2}}} = \frac{\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}}$
  We now only need to multiply top and bottom by $\sqrt{1+\cos\theta}$ to give
  $\frac{\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}} \cdot \frac{\sqrt{1+\cos\theta}}{\sqrt{1+\cos\theta}} = \frac{\sqrt{1-\cos^2\theta}}{1+\cos\theta} = \frac{\sin\theta}{1+\cos\theta}$

  (b) $\sin 3\theta - \sin\theta = 2\cos 2\theta \sin\theta$
  Solution:
  $\sin 3\theta - \sin\theta = \sin(2\theta+\theta) - \sin\theta = \sin 2\theta\cos\theta + \sin\theta\cos 2\theta - \sin\theta$
  $= 2\sin\theta\cos\theta\cos\theta + \sin\theta\cos 2\theta - \sin\theta = \sin\theta(2\cos^2\theta + \cos 2\theta - 1)$
  $= \sin\theta(2\cos^2\theta - 1 + \cos 2\theta) = \sin\theta(\cos 2\theta + \cos 2\theta) = 2\cos 2\theta\sin\theta$
  as required.