EMT 111/112 Solutions to Practice Test I

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Please Note I'm still working on these solutions. Check 1(g) for something that we overlooked in yesterday's class.

1. Solve the inequality.

   (a) $(x+1)(3x-1) \leq 0$
   Solution: By inspection we see that the inequality holds for
   $-1 \leq x \leq \frac{1}{3} \Rightarrow$ the solution set is $[-1, \frac{1}{3}]$

   (b) $\frac{4x}{2x+3} \geq 0$
   Solution: By inspection we see that the inequality is valid
   if $x < -\frac{3}{2}$ or if $x \geq 0 \Rightarrow$ The solution set is $(-\infty, -\frac{3}{2}) \cup [0, +\infty)$

   (c) $\frac{1}{1-x} \leq \frac{3}{x}$
   Solution: Subtracting the RHS from both sides of the inequality we get
   $\frac{1}{1-x} - \frac{3}{x} \leq 0$
   $\Rightarrow \frac{x-3(1-x)}{x(1-x)} \leq 0 \Rightarrow \frac{4x-3}{x(1-x)} \leq 0$
   Our 3 magic numbers are therefore $0, \frac{3}{4}$ and 1.
   By inspection we find that the inequality is valid only for
   $0 < x \leq \frac{3}{4}$ and $x > 1$.
   Therefore the solution set is $(0, \frac{3}{4}] \cup (1, +\infty)$.

   (d) $x^2 < 4$
   Solution: we have $x^2-4 < 0$ and factoring we get $(x-2)(x+2) < 0$.
   Now this inequality is valid only for $-2 < x < 2$.
   Therefore the solution set is $(-2, 2)$.

   (e) $x^3 > x$
   Solution: We have $x^3 - x > 0 \Rightarrow x(x-1)(x+1) > 0$.

Now this inequality is valid only for $-1 < x < 0$ and $x > 1$.
Therefore the solution set is $(-1, 0) \cup (1, +\infty)$.

(f) $|5x - 2| < 6$
Solution: This means that $-6 < 5x - 2 < 6$. From which we get

$$-4 < 5x < 8$$

$$-\frac{4}{5} < x < \frac{8}{5}$$

Which means that the solution set is $(-\frac{4}{5}, \frac{8}{5})$.

(g) $\frac{1}{|x+7|} > 2$
Solution: Multiply both sides of the inequality by $|x + 7|$ to get $1 > 2|x + 7|$. This is equivalent to $|x + 7| < \frac{1}{2}$.
Therefore $-\frac{1}{2} < x + 7 < \frac{1}{2}$ and so $-\frac{15}{2} < x < -\frac{13}{2}$.
However, from the stated problem we see that $x \neq -7$
(since division by zero is not allowed).
Using interval notation , we therefore write the solution set as $(-\frac{15}{2}, -7) \cup (-7, -\frac{13}{2})$.

2. Solve for x: $|2x + 1| = |x - 2|$
Solution: Here we must have either $2x + 1 = x - 2$ or $2x + 1 = 2 - x$.
Solving these two equations we get $x = -3$ or $x = \frac{1}{3}$.
So the solution set is $\{-3, \frac{1}{3}\}$.

3. Find the gradient of the line through $(1, 4)$ and $(-2, 0)$.
Solution: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-4}{-2-1} = \frac{4}{3}$

4. Find the equation of the line that satisfies the given conditions.

(a) through $(3, -1)$; gradient -2
Solution: Here $y = -2x - (-2)(3) + (-1) = -2x + 5$. So the required equation is $y = -2x + 5$.

(b) x-intercept -2; y-intercept 5
Solution: Here the gradient is $\frac{5}{2}$ and $c = 5$. Therefore,
$y = \frac{5}{2}x + 5$ is the required equation.

(c) y-intercept 2; parallel to $x + 2y - 3 = 0$
Solution: Here $m = -\frac{1}{2}$ and $c = 2$. Therefore,
$y = -\frac{1}{2}x + 2$ is the required equation.

(d) through $(1,1)$; perpendicular to $x+5y+8=0$
Solution: This means we have gradient 5,since a perpendicular line has gradient $-\frac{1}{5}$. Our desired equation is therefore
$y = 5x - 5(1) + 1$ or $y = 5x - 4$.

5. Draw the following graphs.

   (a) $2x - y = 1$

   (b) $3x - y + 5 = 0$

   (c) $x = -1$

6. Factor the expression completely.

   (a) $6x^2 + x - 12$
   Solution: $6x^2+x-12 = 6x^2+9x-8x-12 = 3x(2x+3)-4(2x+3)$
   $= (3x-4)(2x+3)$

   (b) $t^3 - 2t^2 - t + 2$
   Solution: $t^3 - 2t^2 - t + 2 = t^2(t-2) - (t-2) = (t-2)(t^2-1)$
   $= (t-2)(t-1)(t+1)$

   (c) $a^2y - b^2y$
   Solution: $a^2y - b^2y = y(a^2 - b^2) = y(a-b)(a+b)$

7. Solve the equation for the indicated variable.

   (a) $F = G\frac{mM}{r^2}$; for r

   (b) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$; for R_1

8. Simplify.

   (a) $\left(\frac{-2x^{\frac{1}{3}}}{y^{\frac{1}{2}}z^{\frac{1}{6}}}\right)^4$

   (b) $\left(\frac{3a^{-2}}{4b^{-\frac{1}{3}}}\right)^{-1}$

9. Find the equation of the circle that satisfies the given conditions.

   (a) center $(1,-3)$; radius 1
   Solution: $(x-1)^2 + (y+3)^2 = 1$.

(b) center $(-1,0)$; passes through $(-2,3)$
Solution: $(x+1)^2+y^2=(-1-(-2))^2+(0-3)^2$
Therefore $(x+1)^2+y^2=10$ is the required equation.

10. Find the center and radius of the given circle.

(a) $x^2+y^2-8x+2y+1=0$
Solution: Rewriting we get $(x-4)^2+(y+1)^2=-1+16+1$ or $(x-4)^2+(y+1)^2=16$. This tells us that the center is $(4,-1)$ and the radius is 4.

(b) $3x^2+3y^2+6x-18y-36=0$
Solution:Dividing by 3 we get $x^2+y^2+2x-6y=12$. Completing the square we get $(x+1)^2+(y-3)^2=12+1+9=22$
Hence $(-1,3)$ is the center and $\sqrt{22}$ is the radius of the circle.

11. Solve for x.

(a) $10^{-x}=2$
Solution: Taking log of both sides we get $-x=\log 2$
So $x=-\log 2=-0.301$

(b) $4+3^{5x}=8$
Solution: Subtracting 4 from both sides we get $3^{5x}=4$
Now taking logs of both sides we get $5x(\log 3)=\log 4$
So $x=\frac{\log 4}{5\log 3}=0.252$

(c) $e^{3-5x}=16$

(d) $x^2 2^x-2^x=0$

(e) $e^{2x}-e^x-6=0$

(f) $\log x=-2$

(g) $\log(3x+5)=2$

(h) $\log_3(2-x)=3$

(i) $\log_2 3+\log_2 x=\log_2 5+\log_2(x-2)$