EMT112 Practice Problems II

Applications - Quadratics,Surds,Indices,Logarithms

September 13, 2010

1. The temperature, H, in ^oF, of a cup of coffee t hours after it is set out to cool is given by the equation:

$$H = 70 + 120(1/4)^t$$

   (a) What is the coffee's temperature initially(that is, at time $t = 0$)? After 1 hour? 2 hours?

   (b) How long does it take the coffee to cool down to 90^oF? 75^oF?

2. Police use the formula $s = \sqrt{30fd}$ to estimate the speed s (in mph) at which a car is traveling if it skids d feet after the brakes are applied suddenly. The number f is the coefficient of friction of the road, which is a measure of the "slipperiness" of the road. The table below gives some typical estimates for f.

	Tar	Concrete	Gravel
Dry	1.0	0.8	0.2
Wet	0.5	0.4	0.1

   (a) If a car skids 65 ft on wet concrete, how fast was it moving when the brakes were applied?

   (b) If a car is traveling at 50 mph, how far will it skid on wet tar?

3. A 15-gram sample of radioactive iodine decays in such a way that the mass remaining after t days is given by $m(t) = 15e^{-0.087t}$ where $m(t)$ is measured in grams. After how many days is there only 5 g remaining?

4. A wooden artifact from an ancient tomb contains 65% of the carbon-14 that is present in living trees. How long ago was the artifact made? Use $Q = Q_0e^{-kt}$ and the fact that the half-life of carbon-14 is 5730 years(Q_0 would be the initial amount of carbon-14, Q the amount of carbon-14 remaining after t years and k the decay constant).

5. The velocity of a sky diver t seconds after jumping is given by $v(t) = 80(1 - e^{-0.2t})$. After how many seconds is the velocity 70 ft/s?

6. The pH of a solution is defined by $pH = -\log[H^+]$. Here $[H^+]$ is the concentration of hydrogen ions measured in moles per liter(M). Solutions with a pH of 7 are defined as neutral, those with $pH < 7$ are acidic and those with $pH > 7$ are basic.

   (a) Calculate the pH of lemon juice which has $[H^+] = 5.0 \times 10^{-3} M$.

   (b) An unknown substance has a hydrogen ion concentration of $[H^+] = 3.1 \times 10^{-8} M$. Find the pH and classify the substance as acidic or basic.

   (c) A certain beer has a pH of 4.6. Find it's hydrogen ion concentration.

7. The American geologist Charles Richter (1900-1984) defined the magnitude of an earthquake to be $M = \log \frac{I}{S}$, where I is the intensity of the earthquake and S is the intensity of a "standard" earthquake.

   (a) If one earthquake is 20 times as intense as another, how much larger is its magnitude on the Richter scale?

   (b) The 1906 earthquake in San Francisco had a magnitude of 8.3 on the Richter scale. How many times more intense was this than the 1995 earthquake in Kobe, Japan (magnitude 7.2)?

8. A farmer has 560 feet of fencing.

   (a) If he encloses a rectangular plot of land of area 19200 sqft, find the dimensions of the land?

   (b) What is the largest rectangular area that a fencing of 560 ft can enclose?

9. Find two positive numbers whose sum is 100 and the sum of whose squares is a minimum.

10. A student makes and sells necklaces at a booth during Amerindian month. The material for each necklace costs her \$600 and she has been selling about 20 per day at \$1000 each. She has been wondering whether or not to raise the price, so she takes a survey and finds that for every \$100 increase she would lose only one sale a day. What price should she set for the necklaces to maximize her profit?