University of Guyana
Faculty of Technology

EMT 121 - Test II

May 4,2012

DIRECTIONS: Answer all questions. NO CALCULATORS ALLOWED.

1. For $A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 3 \\ 7 & -1 & 8 \end{pmatrix}$, $B = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$
   Find $2B - AB$ (8 marks)

   $$2B - AB = \begin{pmatrix} 6 \\ -4 \\ -2 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 15 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \\ -17 \end{pmatrix}$$

2. Find the area of the triangle with vertices (0,0),(3,5) and (5,3).
   (8 marks)

   $$A = \pm\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 3 & 5 & 1 \\ 5 & 3 & 1 \end{vmatrix} = 8$$

3. A matrix is said to be orthogonal if $A^TA = I$.Thus the inverse of an orthogonal matrix is just its transpose. What are the possible values of det(A) if A is an orthogonal matrix? (10 marks)

   $det(A^TA) = det(I) = 1 \Rightarrow det(A^T) \cdot det(A) = 1$
   but $det(A) = det(A^T) \Rightarrow (det(A))^2 = 1 \Rightarrow det(A) = \pm 1$

4. For

   $$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 8 & 7 \\ 3 & 6 & 7 & 6 \\ 5 & 10 & 15 & 25 \end{pmatrix}$$

find det(A) . (8 marks)

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 8 & 7 \\ 3 & 6 & 7 & 6 \\ 5 & 10 & 15 & 25 \end{vmatrix} \to \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & -2 & -6 \\ 0 & 0 & 0 & 5 \end{vmatrix} \Rightarrow \det(\mathrm{A}) = 1{\cdot}1{\cdot}{-2}{\cdot}5 = -10$$

5. Solve the given system of linear equations.You can use any method.

$$-x_1 + x_2 + 2x_3 = 1$$
$$2x_1 + 3x_2 + x_3 = -2$$
$$x_1 + 4x_2 + 3x_3 = -1$$

(10 marks)

$$\begin{pmatrix} 1 & 4 & 3 & -1 \\ 2 & 3 & 1 & -2 \\ -1 & 1 & 2 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 4 & 3 & -1 \\ 0 & -5 & -5 & 0 \\ 0 & 5 & 5 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 4 & 3 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Let $x_3 = t$, then $x_2 = -t$ and $x_1 = -1 + 4t - 3t = t - 1$, where t is any real number.

6. Determine if $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(k+1)(k+3)}$ converges or diverges. If it converges find the sum. (8 marks)

The above series is a telescoping series and therefore it converges.

$\displaystyle\sum_{k=1}^{\infty} \frac{1}{(k+1)(k+3)} = \frac{1}{2}\left(\sum_{k=1}^{\infty} \frac{1}{k+1} - \frac{1}{k+3}\right)$

$\Rightarrow S_n = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \ \frac{1}{5} + \frac{1}{4} - \frac{1}{6} ... + \frac{1}{n+1} - \frac{1}{n+3}\right)$

$\displaystyle\lim_{n\to\infty} S_n = \frac{1}{2}\lim_{n\to\infty}\left(\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \frac{1}{5} - ... - \frac{1}{n+2} - \frac{1}{n+3}\right)$

$\displaystyle = \frac{1}{2}\lim_{n\to\infty}\left(\frac{5}{6} - \frac{1}{n+2} - \frac{1}{n+3}\right) = \frac{5}{12}$

$\Rightarrow$ the sum of the series is $\frac{5}{12}$

7. Find the radius of convergence for $\displaystyle\sum_{n=0}^{\infty}\left(\frac{x}{3}\right)^n$. (8 marks)

$$R = \lim_{n\to\infty}\left|\frac{(1/3)^n}{(1/3)^{n+1}}\right| = 3$$

8. Write down the Taylor series for $\cos x$ centered at 0. (8 marks)

   $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + ... = \displaystyle\sum_{k=0}^{\infty}(-1)^k \frac{x^{2k}}{(2k)!}$

9. Use summation notation to write down the summation of the first 10 positive integers. (8 marks)

   $\displaystyle\sum_{i=1}^{10} i$

10. Write down the number that represents the sum of the first 10000 odd numbers. (8 marks)

    The number is 10000^2 or 100,000,000.

11. Use DeMoivre's Theorem to give a formula for $\cos 3\theta$. (8 marks)

    $\cos 3\theta + j \sin 3\theta = (\cos\theta + j\sin\theta)^3 \Rightarrow \cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$

12. Write the complex number $1 + j$ in polar form. (8 marks)

    $r = \sqrt{1^2 + 1^2} = \sqrt{2}$, $\theta = \tan^{-1}(\frac{1}{1}) = \frac{\pi}{4}$
    $\Rightarrow \sqrt{2}\,(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4})$ is the polar form of $1 + j$.