University of Guyana
Faculty of Technology

EMT 121 - Practice Final with solutions

May 31, 2012

1. Use Boolean algebra to simplify the following expressions, then draw logic circuits for the simplified expressions:

   (a) $A(B + AB) + AC$

   (b) $(A + B)(\bar{A} + \bar{B})$

   (c) $\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C$

   (a) $A(B + AB) + AC = AB + AC = A(B + C)$
   (b) $(A + B)(\bar{A} + \bar{B}) = A\bar{A} + A\bar{B} + \bar{A}B + B\bar{B}$

   $$= 0 + A\bar{B} + \bar{A}B + 0 = A\bar{B} + \bar{A}B = A \oplus B$$

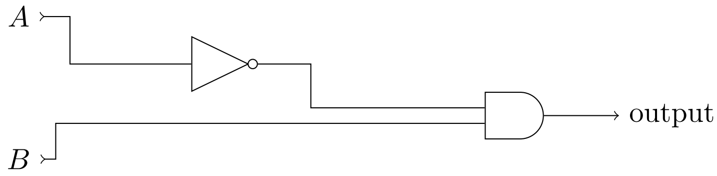
   (c) $\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C$

   $$= \bar{A}\bar{B}(\bar{C} + C) + A\bar{B}(\bar{C} + C) = \bar{A}\bar{B} + A\bar{B} = \bar{B}(\bar{A} + A) = \bar{B}$$

2. Given the following truth table write a corresponding Boolean expression and draw a logic circuit capable of producing the required outputs.

Input A	*Input* B	*Input* C	*Output* X
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

   Using SOP we get $\bar{A}B\bar{C} + \bar{A}BC = \bar{A}B(\bar{C} + C) = \bar{A}B$



3. Determine whether the series $\sum_{n=1}^{\infty} 2^n$ converges or diverges.

   By the Divergence test the series diverges since $\lim_{n\to\infty} 2^n = \infty \neq 0$.

4. Find the Maclaurin series for $f(x) = e^{x^2}$.

   No need to do this one from scratch. Here we can use the fact that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ and substitute x^2 for x to get $e^{x^2} = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$

5. Find the radius of convergence of the power series

   $$\sum_{n=0}^{\infty} \frac{(-1)^n (x+1)^n}{2^n}$$

   $R = \lim_{n\to\infty} \left| \frac{(-1)^n}{2^n} \cdot \frac{2^{n+1}}{(-1)^{n+1}} \right| = \lim_{n\to\infty} 2 = 2$
   Since the center of the series is -1, it means that the series converges in the interval $(-3, 1)$.

6. The roots of the quadratic equation $x^2 - 4x + c = 0$ are the complex numbers $2 + j$ and $2 - j$. Find the value of the constant c.

   There are a couple of ways to get this but if we do the multiplication $(x - 2 - j)(x - 2 + j)$ shouldn't we get the result?

7. The position vectors of two points A and B are $2i + 3j$ and $3i - 8j$ respectively. D is the midpoint of AB and the point E divides OD in the ratio 2:3. Find the position vector of E.

   Since D is the midpoint of AB its coordinates are (5/2,-5/2) and hence its position vector , $\overrightarrow{OD}$ is $\frac{5}{2}i - \frac{5}{2}j \Rightarrow \overrightarrow{OE} = \frac{2}{5}\overrightarrow{OD} = i - j$

8. The first four terms of an AP are 2,5,$(2x + y + 7)$ and $(2x - 3y)$ respectively where x and y are constants. Find the value of x and the value of y.

   Here the common difference is 3 and we have 2 equations to solve simultaneously, $2x + y + 7 = 8$ and $2x - 3y = 11$. Solving these will give $x = 7/4$ and $y = -5/2$.

9. (a) Find the sum to n terms of the geometric series

$$4 + 2 + 1 + \frac{1}{2} + \cdots$$

For this geometric series $r = 1/2$. This means that
$S_n = \frac{4(1-(\frac{1}{2})^n)}{1-\frac{1}{2}} = 8(1 - (\frac{1}{2})^n)$

(b) Deduce the sum to infinity of the series.
The sum to infinity is $\lim_{n\to\infty} 8\left(1 - (\frac{1}{2})^n\right) = 8$

10. Three points A, B and C have coordinates (1,2), (2,5) and (0,-4) respectively relative to the origin O.

(a) Express the position vector of EACH of A,B and C in terms of i and j.
(a) $\overrightarrow{OA} = i + 2j$, $\overrightarrow{OB} = 2i + 5j$, and $\overrightarrow{OC} = -4j$

(b) If $\overrightarrow{AB} = \overrightarrow{CD}$, find the position vector of D in terms of i and j.
$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD} = \overrightarrow{OC} + \overrightarrow{AB} = \overrightarrow{OC} + \overrightarrow{AO} + \overrightarrow{OB}$
$= -4j - i - 2j + 2i + 5j = i - j$

11. Find the values of θ $(0 \leq \theta \leq 2\pi)$ for which the vectors $\cos\theta i + \sqrt{3}j$ and $\frac{1}{4}i + \sin\theta j$ are parallel.
If the two vectors are parallel then the following equation is valid.

$$\frac{\sqrt{3}}{\cos\theta} = \frac{\sin\theta}{1/4}$$

From which we get (with steps deliberately left out)

$$\sin 2\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow 2\theta = \sin^{-1}\frac{\sqrt{3}}{2}$$

So $2\theta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$. This means that $\theta = \frac{\pi}{6}$ or $\frac{\pi}{3}$

12. For the triangle whose vertices are $A(-2, 0)$, $B(4, 1)$, and $C(5, 4)$.

(a) Find the area.
$A = \pm\frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 4 & 1 & 1 \\ 5 & 4 & 1 \end{vmatrix} = \pm\frac{1}{2}(-2\times\ (1\times1-4\times1)+1\times(4\times4-5\times1))$
$= \pm\frac{1}{2}(6 + 11) = \frac{17}{2}$ or $8\frac{1}{2}$

(b) Find $\cos A$, $\cos B$, and $\cos C$.
$\cos A = \frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{\parallel \overrightarrow{AB} \parallel \parallel \overrightarrow{AC} \parallel} = \frac{<6,1>\cdot<7,4>}{\sqrt{37}\sqrt{65}} = \frac{46}{\sqrt{37}\sqrt{65}}$
similarly,
$\cos B = \frac{-9}{\sqrt{37}\sqrt{10}}$ and $\cos C = \frac{19}{\sqrt{65}\sqrt{10}}$.

13. Given the points A(1,2) and B(-1,3) , find

(a) the coordinates of M the midpoint of AB.
M has coordinates $(0, 5/2)$.

(b) the equation of the line through the origin parallel to AB.
The line through the origin and $\parallel$ to AB is given by $y = mx$ where $m = -1/2$, the gradient of AB.

14. In a triangle ABC show that $\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AD}$ where D is the midpoint of BC.

Since D is the midpoint of BC
$\Rightarrow 2\overrightarrow{BD} = \overrightarrow{BC}$
$\Rightarrow 2(\overrightarrow{BA} + \overrightarrow{AD}) = \overrightarrow{BA} + \overrightarrow{AC}$
$\Rightarrow 2\overrightarrow{AD} = -2\overrightarrow{BA} + \overrightarrow{BA} + \overrightarrow{AC}$
$\Rightarrow 2\overrightarrow{AD} = -\overrightarrow{BA} + \overrightarrow{AC}$
$\Rightarrow 2\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{AC}$ as required.

15. (a) Express the complex number $z = \frac{11-2j}{3+4j}$ in the form $a + jb$ where a and b are real numbers.

Multiplying both numerator and denominator by $3 - 4j$ we get
$z = \frac{(11-2j)(3-4j)}{(3+4j)(3-4j)} = \frac{33-44j-6j-8}{25} = \frac{25-50j}{25} = 1 - 2j$

(b) Hence express z^2 and jz in a similar form.

$z^2 = (1 - 2j)^2 = -3 - 4j$ and $jz = j(1 - 2j) = 2 + j$

(c) Find the modulus of z.
$|z| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$