University of Guyana
Faculty of Technology

EMT 121 - Practice Final

May 23, 2012

1. Use Boolean algebra to simplify the following expressions, then draw logic circuits for the simplified expressions:

   (a) $A(B + AB) + AC$

   (b) $(A + B)(\bar{A} + \bar{B})$

   (c) $\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C$

2. Given the following truth table write a corresponding Boolean expression and draw a logic circuit capable of producing the required outputs.

Input A	*Input* B	*Input* C	*Output* X
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

3. Determine whether the series $\sum_{n=1}^{\infty} 2^n$ converges or diverges.

4. Find the Maclaurin series for $f(x) = e^{x^2}$.

5. Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x+1)^n}{2^n}$$

6. The roots of the quadratic equation $x^2 - 4x + c = 0$ are the complex numbers $2 + j$ and $2 - j$. Find the value of the constant c.

7. The position vectors of two points A and B are $2i + 3j$ and $3i - 8j$ respectively. D is the midpoint of AB and the point E divides OD in the ratio 2:3. Find the position vector of E.

8. The first four terms of an AP are 2,5,$(2x + y + 7)$ and $(2x - 3y)$ respectively where x and y are constants. Find the value of x and the value of y.

9. (a) Find the sum to n terms of the geometric series

$$4 + 2 + 1 + \frac{1}{2} + \cdots$$

   (b) Deduce the sum to infinity of the series.

10. Three points A, B and C have coordinates (1,2), (2,5) and (0,-4) respectively relative to the origin O.

   (a) Express the position vector of EACH of A,B and C in terms of i and j.

   (b) If $\overrightarrow{AB} = \overrightarrow{CD}$, find the position vector of D in terms of i and j.

11. Find the values of θ $(0 \leq \theta \leq 2\pi)$ for which the vectors $\cos\theta i + \sqrt{3}j$ and $\frac{1}{4}i + \sin\theta j$ are parallel.

12. For the triangle whose vertices are $A(-2, 0)$, $B(4, 1)$, and $C(5, 4)$.

   (a) Find the area.

   (b) Find $\cos A$, $\cos B$, and $\cos C$.

13. Given the points A(1,2) and B(-1,3) , find

   (a) the coordinates of M the midpoint of AB.

   (b) the equation of the line through the origin parallel to AB.

14. In a triangle ABC show that $\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AD}$ where D is the midpoint of BC.

15. (a) Express the complex number $z = \frac{11-2j}{3+4j}$ in the form $a + jb$ where a and b are real numbers.

   (b) Hence express z^2 and jz in a similar form.

   (c) Find the modulus of z.

16. Solve the given matrix equation for a, b,c, and d.

$$\begin{pmatrix} a-b & 2b+c \\ c-2d & a+d \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 5 & -2 \end{pmatrix}$$

17. Find the inverse of

$$\begin{pmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \\ -2 & 0 & 3 \end{pmatrix}$$

18. Convert 0x38F to its octal equivalent.

19. In a sixteen-bit digital system, where all numbers are represented in two's complement form, what is the largest (most positive) quantity that may be represented with those sixteen bits? What is the smallest (most negative) quantity that may be represented? Express your answers in both binary (two's complement) and decimal form.