

University of Guyana  
Faculty of Technology

EMT 121  
PROBLEM SET III (ARITHMETIC AND GEOMETRIC SERIES)  
ANSWERS

February 17, 2011

1. If air resistance is neglected, a falling object travels 16 ft during the first second, 48 ft during the next, 80 ft during the next, and so on. These distances form the arithmetic sequence 16, 48, 80, . . .
  - (a) Find a formula for the  $n^{\text{th}}$  term in the sequence of distances. Calculate the fifth and tenth terms.
  - (b) Calculate  $S_1, S_2$ , and  $S_3$ , the total distance an object falls in 1, 2, and 3 seconds respectively.
  - (c) Give a formula for  $S_n$ , the distance fallen in  $n$  seconds.

Ans. (a)  $a_n = 32n - 16$ ,  $a_5 = 144$ ,  $a_{10} = 304$  (b)  $S_1 = 16$ ,  $S_2 = 64$ ,  $S_3 = 144$  (c)  $S_n = 16n^2$

2. Expand the sums.

(a)  $\sum_{i=1}^5 i^2 = 1 + 4 + 9 + 16 + 25$

(b)  $\sum_{i=10}^{20} (i+1)^2 = 121 + 144 + 169 + 196 + 225 + 256 + 289 + 324 + 361 + 400 + 441$

(c)  $\sum_{n=1}^7 (-1)^{n-1} 2^n = 2 - 4 + 8 - 16 + 32 - 64 + 128$

3. Write each sum using sigma notation.

(a)  $3+6+9+12+15+18+21=3\sum_{i=1}^7 i$

(b)  $30+25+20+15+10+5=5\sum_{i=1}^6 (7-i)$

$$(c) \ 1/2+1+3/2+2+5/2+3+7/2+4 = \sum_{i=1}^8 i/2$$

4. Find the sum of the first nine terms of the series:  $7+14+21+ \dots$

$$\text{Ans. } S_9 = 7 \sum_{i=1}^9 i = 7 \frac{(1+9)}{2} \cdot 9 = 315$$

5. Find the sum of the geometric series  $\sum_{i=0}^{17} 7(-z)^i$

$$\text{Ans. } S = \frac{7(1+z^{18})}{1+z}$$

6. A patient is given a 20mg injection of a therapeutic drug. Each day, the patient's body metabolizes 50% of the drug present, so that after 1 day only half of the original amount remains, after 2 days only one-fourth remains, and so on. The patient is given a 20 mg injection of the drug every day at the same time.

- (a) Write a geometric series that gives the drug level in this patient's body right after the  $n^{\text{th}}$  injection.

$$\text{Ans. } 20 \sum_{i=1}^n (1/2)^{i-1}$$

- (b) What quantity of the drug remains in the patient's body after the  $10^{\text{th}}$  injection.

$$\text{Ans. } S_{10} = \frac{20(1-(1/2)^{10})}{1-1/2} = 39.961 \text{ mg}$$

7. Worldwide consumption of oil was 27.5 billion barrels in 2001. Assume that consumption continues to increase at 1.2% per year, the rate for the previous decade.

- (a) Write a sum representing the total oil consumption between the start of 2001 and the end of 2025.

$$\text{Ans. } 27.5 \sum_{i=1}^{25} (1.012)^{i-1}$$

- (b) Evaluate this sum.

$$\text{Ans. } 796,240,677,288 \text{ barrels.}$$