Continuation of 20/11/2009 class on Limits

Laurel Benn

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These notes are a continuation of the lecture notes given on 20/11/2009.

Limit Laws

Suppose $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist and c is a real constant. Then

1. $\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
2. $\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
3. $\lim_{x\to a} cf(x) = c\lim_{x\to a} f(x)$
4. $\lim_{x\to a}(f(x)\cdot g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$
5. $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, provided $\lim_{x\to a} g(x) \neq 0$.
6. $\lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n$
7. $\lim_{x\to a} c = c$
8. $\lim_{x\to a} x = a$
9. $\lim_{x\to a} x^n = a^n$
10. $\lim_{x\to a} \sqrt[n]{a} = \sqrt[n]{a}$, where n is a positive integer.
11. $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$, where n is a positive integer.

Direct Substitution Property

If f is a polynomial or rational function, and a is in the domain of f, then $\lim_{x \to a} f(x) = f(a)$

Example 1 *Show that* $\lim_{x \to 1} \frac{\sqrt{x^2+3}}{x} = 2.$

$$\lim_{x \to 1} \frac{\sqrt{x^2+3}}{x} = \frac{\sqrt{\lim_{x \to 1}(x^2+3)}}{\lim_{x \to 1} x} = \frac{\sqrt{4}}{1} = 2$$

Squeeze Theorem

Suppose $f(x) \leq g(x) \leq h(x)$, for all x in an interval containing $x = a$. Then if $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} h(x) = L$ then $\lim_{x \to a} g(x) = L$

Example 2 *Show that* $\lim_{x \to 0} x^2 \cos(\frac{1}{x}) = 0$
Since values of cos$(\frac{1}{x})$ *lies between -1 and 1, it follows that*
$-x^2 \leq x^2 \cos(\frac{1}{x}) \leq x^2$
but $\lim_{x \to 0} -x^2 = 0$ *and* $\lim_{x \to 0} x^2 = 0$
$\Rightarrow$ *by the Squeeze Theorem* $\lim_{x \to 0} x^2 \cos(\frac{1}{x}) = 0.$

Exercises

Evaluate.

1. $\lim_{x \to -2} \frac{x^2+x-2}{x-1}$. Ans. 0

2. $\lim_{x \to 1} \frac{x^3-1}{x-1}$. Ans. 3

3. $\lim_{x \to -3} \frac{x^2+x-6}{x+3}$. Ans. -5